TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 2   2020

Section 1   Calculator Free
Sequences

STUDENT'S NAME ____

DATE: Friday 22nd May   **TIME:** 25 minutes   **MARKS**: 24

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

A sequence is such that its' first 3 terms are 20, 30, 45…

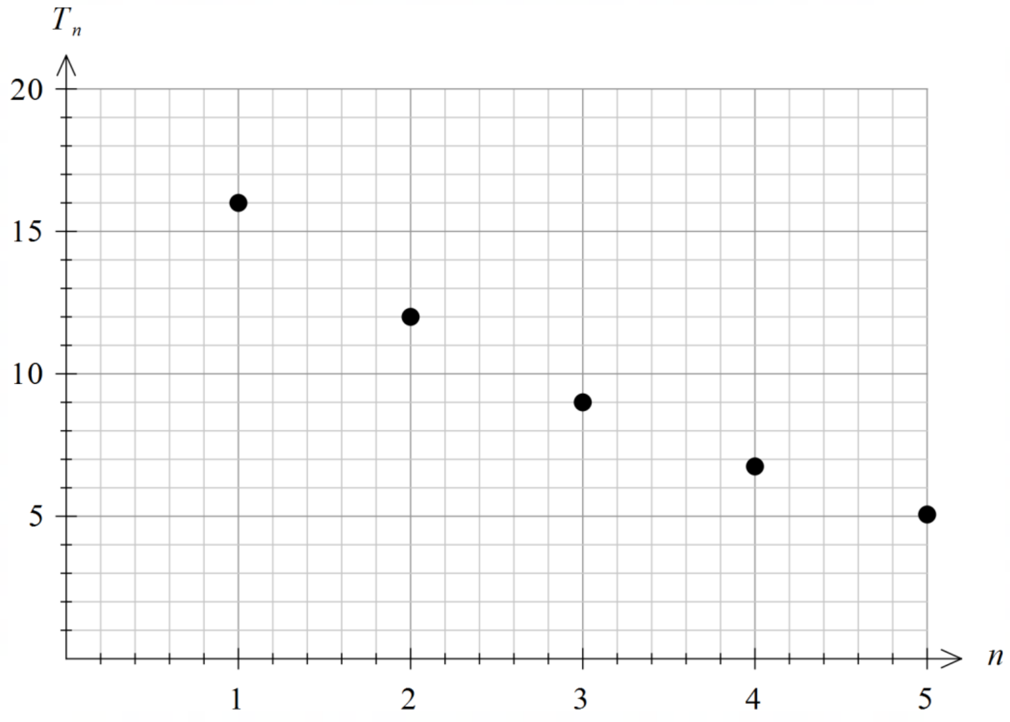
(a) **Show** that the sequence is geometric. [2]

(b) Write the recursive rule for the sequence. [2]

(c) As the value of n continues to increase, will the value of the term in the sequence increase, decrease or reach a steady state? [1]

2. (5 marks)

On the graph below a geometric sequence, T_n, has been plotted.



(a) For the geometric sequence, determine:

(i) The simplified ratio [1]

(ii) The general rule [1]

(b) Add the first five terms of the arithmetic sequence, $U_{n+1} = U_n + 2$, $U_1 = 5$ to the graph above. [2]

(c) Determine the values of n for which $U_n > T_n$. [1]

3. (8 marks)

An arithmetic sequence has a fourth term of -3 and a tenth term of 51.

(a) Determine the rule for the n^{th} term of the sequence. [3]

(b) Calculate the 21^{st} term of the sequence. [2]

(c) State the last term of the sequence which has a value less than 200. [3]

4. (6 marks)

A wetland has a population of black-necked storks is initially at 20 and has a natural decrease of half the population per year. At the end of each year, 8 extra black-necked storks are introduced to the wetlands.

(a) Determine the recursive rule for the population of black-necked storks. [2]

(b) Determine the long-term steady state of the black-necked stork population. [2]

(c) The wetlands would like to maintain a population of 24 black-necked storks. What is the yearly addition of black-neck storks required to produce this steady state given that all other conditions are to remain the same? [2]

TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 2 2020

Section 2 Calculator Assumed
Sequences

STUDENT'S NAME ____

DATE: Friday 22nd May **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (8 marks)

A sequence is defined by the recurrence relation:

$T_{n+2} = a.T_{n+1} + b.T_n$ where $T_1 = x$ and $T_2 = y$.

(a) Determine the first five terms of the sequence if $a = -1$, $b = 1$, $x = 2$ and $y = 3$. [3]

(b) The sequence 10, 20, 70, 200, 610 obeys the recurrence relation given. By using simultaneous equations or otherwise determine the values of *a* and *b* and hence state the recursive rule. [5]

6. (7 marks)

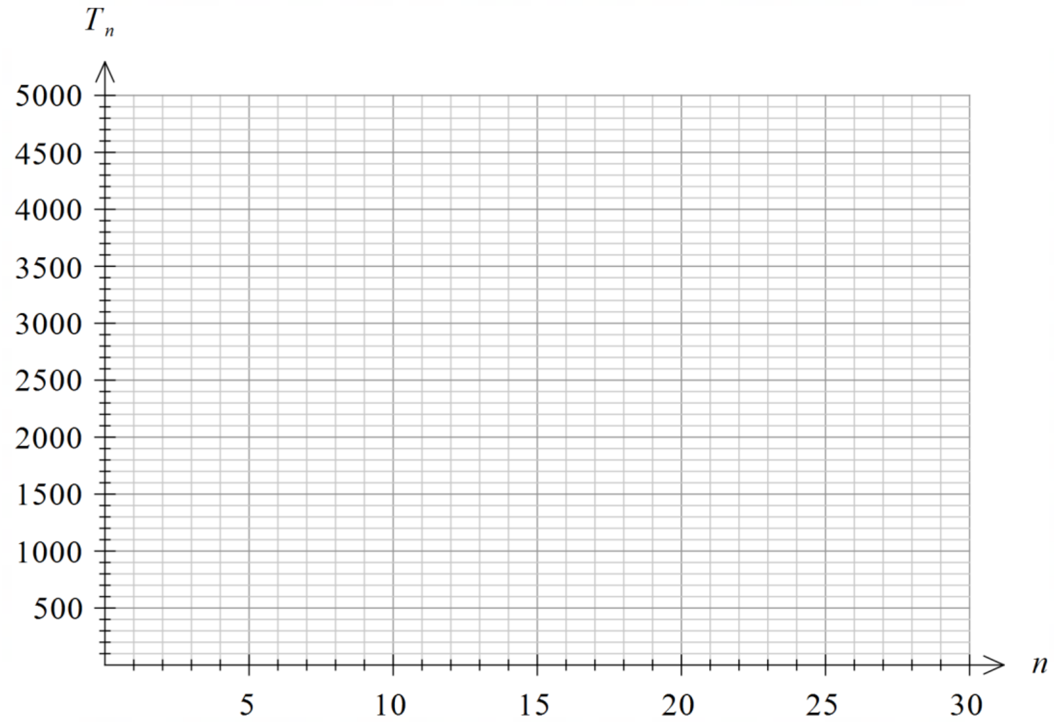
A plantation has 4 800 trees. The plantation manager is interested in modelling what would happen if each year, 10% of the existing trees were cut down for timber and another 250 new trees are planted.

The number of trees, T_n, at the start of year n can be modelled by $T_{n+1} = 0.9T_n + 250$, $T_1 = 4800$.

(a) Use the recurrence relation to complete the missing values in the following table. [2]

n	5	10	15	20	25	30
T_n	4009					2608

(b) Plot the values from the table in part (a) on the axes below. [2]



(c) Comment on how the number of trees in the plantation is changing. [1]

(d) Does the model predict that eventually there will be no trees left in the plantation? Justify your answer algebraically. [2]

7. (7 marks)

In ideal conditions a certain bacterium double their numbers every 15 minutes. A raw fish, under these conditions, with 180 harmful bacteria is left lying on a kitchen bench.

(a) Show that the ratio at which the bacteria increase every hour is 16. [1]

(b) State the recursive rule for the number of bacteria, where *n* is the number of hours the fish has been on the counter. [2]

(c) Estimate the number of bacteria on the fish after 2 hours. [1]

After being left on the counter for 2 hours, the fish is cooked in an oven, where the bacteria are killed at a rate of 78% per hour. The fish is to be cooked for 3 hours

(d) Given that the fish should contain less than 500 bacteria to be considered safe to eat, will the bacteria reduce enough after being cooked for 3 hours for the fish to be eaten safely. Justify your answer. [3]

8. (5 marks)

A sequence is defined by the rule $A_{n+1} = 2 - 3A_n$, where $n = 1, 2, 3, ...$ for $A_1 = k$. Given that k is a constant, determine k if $A_4 = -598$.